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A MICROLOCAL VERSION OF THE RIEMANN-HILBERT CORRESPONDANCE

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1. – Introduction

Let X be a complex n-dimensional manifold. Recall that the "Riemann-Hilbert correspondance" consists of the two following commutative diagrams, together with the assertion that all the arrows are equivalences of categories :

(1.1)
$$\begin{tikzcd} D^b_{\mathbb{C}-c}(X)^\circ \arrow[r, shift left, "RH"] \arrow[rr, bend left, "Rhom(\cdot,\mathcal{O}_X)"] & D^b_{\mathrm{r-h}}(\mathcal{D}_X) \arrow[l, shift left, "\mathrm{Sol}"] \arrow[r, "\mathcal{D}^\infty_X \otimes_{\mathcal{D}_X} (\cdot)"] & D^b_h(\mathcal{D}^\infty_X) \arrow[ll, bend left, "\mathrm{Sol}"] \end{tikzcd}$$

(1.2)
$$\begin{tikzcd} \mathrm{Perv}(X)^\circ \arrow[r, shift left, "RH"] \arrow[rr, bend left, "Rhom(\cdot,\mathcal{O}_X)"] & \mathrm{Reghol}(\mathcal{D}_X) \arrow[l, shift left, "\mathrm{Sol}"] \arrow[r, "\mathcal{D}^\infty_X \otimes_{\mathcal{D}_X} (\cdot)"] & \mathcal{H}\mathrm{ol}(\mathcal{D}^\infty_X) \arrow[ll, bend left, "\mathrm{Sol}"] \end{tikzcd}$$

We make use of the following notations :

$D^b_{\mathbb{C}-c}(X)$ is the derived category of bounded complexes of sheaves of $\mathbb{C}$-vector spaces on X with $\mathbb{C}$-constructible cohomology.

$\mathrm{Reghol}(\mathcal{D}_X)$ is the abelian category of regular holonomic (left) $\mathcal{D}_X$-modules.

$\mathcal{H}\mathrm{ol}(\mathcal{D}^\infty_X)$ is the category of modules of the form $\mathcal{D}^\infty_X \underset{\mathcal{D}_X}{\otimes} \mathcal{M}$ where $\mathcal{M}$ is a holonomic $\mathcal{D}$-module.

$D^b_{\mathrm{r-h}}(\mathcal{D}_X)$ is the derived category of bounded complexes of $\mathcal{D}_X$-modules with regular holonomic cohomology. $D^b_h(\mathcal{D}^\infty_X)$ is the derived catgeory of bounded complexes of admissible $\mathcal{D}^\infty_X$-modules (in the sense of [S-K-K]) with cohomology in $\mathcal{H}\mathrm{ol}(\mathcal{D}^\infty_X)$.

$\mathrm{Perv}(X)$ is the full abelian subcategory of "perverse sheaves" of $D^b_{\mathbb{C}-c}(X)$, where we adopt for our purpose a definition shifted by $n = \dim_{\mathbb{C}} X$ from the usual one, i.e. given $F \in Ob\, D^b_{\mathbb{C}-c}(X)$, we say F is an object of $\mathrm{Perv}(X)$ iff $F[n]$ is perverse in the usual sense of [BBD] (e.g. if $Y \subset X$ is a purely d-codimensional complex set then we say that $\mathbb{C}_Y[-d]$ is perverse ; see § 4).

Recall that one sets $\mathrm{Sol}(\mathcal{M}) = R\mathcal{H}om_{\mathcal{D}}(\mathcal{M},\mathcal{O})$ or $R\mathcal{H}om_{\mathcal{D}^\infty}(\mathcal{M},\mathcal{O})$ accordingly, and that the arrows bearing that name in (1.1) and (1.2) were constructed in [K 1], that the equivalence under $\mathcal{D}^\infty_X \overset{\mathrm{L}}{\otimes}_{\mathcal{D}_X}(\cdot)$ was proven in [K-K] and that the construction of the temperate $R\mathcal{H}om(\cdot,\mathcal{O})$-functor RH and proof that RH is an equivalence was performed in [K 1, 2].

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An independent proof that Sol is an equivalence is performed in [M 1 and 2]. See [B] for a review of these results.

The point of interest here is to give a microlocal version of (1.2). Namely, if $\pi : T^*X \to X$ is the cotangent bundle of X, and $p \in \overset{\circ}{T^*} X \underset{\mathrm{def}}{=} T^*X \backslash T^*_X X$, one has the abelian category $\mathrm{Reghol}(\mathcal{E}_{X,p})$ of germs of regular holonomic modules over the ring of microdifferential operators $\mathcal{E}_{X,p}$ of [S-K-K] which should be equivalent to a category defined by a suitable microlocalization of $\mathrm{Perv}(X)$. The precise statement goes as follows.

We set $\mathbb{C}^\times := \mathbb{C}\backslash\{0\}$ and $\gamma : T^*X \to T^*X/\mathbb{C}^\times$.

THEOREM 1. — *One has the following commutative diagram (1.3) and all the horizontal arrows are equivalences of categories.*

$$
(1.3)\qquad
\begin{tikzcd}[column sep=huge]
\mathrm{Perv}(X;\mathbb{C}^\times p)^\circ \arrow[r, "\mu RH"] \arrow[d] \arrow[rr, bend left, "\gamma^{-1}R\gamma_*\mu hom(\cdot,\mathcal{O}_X)"] & \mathrm{Reghol}(\mathcal{E}_{X,p}) \arrow[r, "\mathcal{E}^\infty_{X,p}\otimes_{\mathcal{E}_{X,p}}(\cdot)"] \arrow[d, "\mathcal{E}^{\mathbb{R},f}_{X,p}\otimes_{\mathcal{E}_{X,p}}(\cdot)"] & \mathcal{H}ol(\mathcal{E}^\infty_{X,p}) \arrow[d, "\mathcal{E}^{\mathbb{R}}_{X,p}\otimes_{\mathcal{E}^\infty_{X,p}}(\cdot)"] \\
\mathrm{Perv}(X;p)^\circ \arrow[r, shift left, "T\text{-}\mu hom(\cdot,\mathcal{O}_X)"] \arrow[rr, bend right, "\mu hom(\cdot,\mathcal{O}_X)_p"'] & \mathrm{Reghol}(\mathcal{E}^{\mathbb{R},f}_{X,p}) \arrow[l, shift left, "\mathrm{Sol}_p"] \arrow[r, "\mathcal{E}^{\mathbb{R}}_{X,p}\otimes_{\mathcal{E}^{\mathbb{R},f}_{X,p}}(\cdot)"] & \mathcal{H}ol(\mathcal{E}^{\mathbb{R}}_{X,p}) \arrow[ll, bend left=60, "\mathrm{Sol}_p"]
\end{tikzcd}
$$

Here :

$\mathcal{E}^\infty_X$ is the sheaf of infinite order microdifferential operators of [S-K-K],

$\mathcal{E}^{\mathbb{R}}_X$ is the sheaf of holomorphic microlocal operators of [S-K-K] and $\mathcal{E}^{\mathbb{R},f}_X$ is its temperate analogue of [A].

An object of $\mathrm{Reghol}\,\mathcal{E}^{\mathbb{R},f}_{X,p}$ is by definition of the form $\mathcal{E}^{\mathbb{R},f}_{X,p} \underset{\mathcal{E}_X}{\otimes} \mathcal{M}$ with $\mathcal{M} \in Ob\,\mathrm{Reghol}\,\mathcal{E}_{X,p}$, with a similar definition for $\mathcal{H}ol(\mathcal{E}^\infty_{X,p})$ and $\mathcal{H}ol(\mathcal{E}^{\mathbb{R}}_{X,p})$.

The categories $\mathrm{Perv}(X;\mathbb{C}^\times p)$ and $\mathrm{Perv}(X;p)$ are defined below.

$\mu\,\mathrm{hom}(\cdot,\cdot)$ is Kashiwara and Schapira's functor of [K-S 2], and $T\text{-}\mu hom(\cdot,\mathcal{O}_X)$ is the temperate version of $\mu\,\mathrm{hom}(\cdot,\mathcal{O}_X)$ of [A], while $\mu RH := \gamma^{-1}R\gamma_*\, T\text{-}\mu hom(\cdot,\mathcal{O}_X)$.

Assuming the definition of $\mathrm{Perv}(X;p)$, the construction of Sol_p is implicit in [K-S 1].

The various microlocalizations of $\mathrm{Perv}(X)$ are performed by essential use of the microlocal theory of sheaves of Kashiwara and Schapira [K-S 2] and by using the microlocal characterisation of perverse sheaves of loc.cit.

We stress the point that these microlocalizations rely on necessary *real* (subanalytic) geometry.

The main tool in the proof is the invariance by canonical transformations which allows one to make use of the generic position theorem of [K-K] which reduces the situation to that of (regular holonomic) $\mathcal{D}$-modules.

2. – The category $D^b_{\mathbb{R}-c}(X;\Omega)$

Let X be a real analytic manifold, $D^b(X)$ the derived category of the category of bounded complexes of sheaves on X and $D^b_{\mathbb{R}-c}(X)$ its full triangulated subcategory of complexes with $\mathbb{R}$-constructible cohomology. The following is detailed in [A, Appendix].

If $\Omega \subset T^*X$ is any subset of the cotangent bundle of X the fundamental category occuring in [K-S 2] is

$$D^b(X;\Omega) \underset{\text{def}}{=} D^b(X)/\mathcal{N}_\Omega$$

where $\mathcal{N}_\Omega$ is the null-system of objects F whose micro-support $SS(F)$ does not meet Ω (cf. loc.cit).

We set here

$$D^b_{\mathbb{R}-c}(X;\Omega) = D^b_{\mathbb{R}-c}(X)/\mathcal{N}_\Omega \cap Ob\, D^b_{\mathbb{R}-c}(X).$$

Note that if $\Omega' \subset \Omega$ there is a canonical functor $D^b_{\mathbb{R}-c}(X;\Omega) \longrightarrow D^b_{\mathbb{R}-c}(X;\Omega')$.

If $\Omega = \{p\}$ is a point we write $D^b(X;p)$ instead of $D^b(X;\{p\})$ and so forth.

By the results of [K-S 2] if is easy to see that

LEMMA 2.1. — *$D^b_{\mathbb{R}-c}(X;p)$ is a full triangulated subcategory of $D^b(X;p)$.*

An adaptation of the microlocal kernel operations of [K-S 2] yields also the invariance under "extended canonical transformations" of loc.cit.

More precisely, let Y be another real manifold and denote by q_j the j-th projection of $X \times Y$ and by $(\cdot)^a$ the antipodal map of T^*Y.

Let $p_X \in T^*X$, $p_Y \in T^*Y$ and $K \in Ob\, D^b_{\mathbb{R}-c}(X \times Y)$ satisfying the following condition :

(2.1) $$SS(K) \cap (\{p_X\} \times T^*Y) \subset \{(p_X, p_Y^a)\} \text{ in the neighborhood of that point.}$$

For $F \in Ob\, D^b_{\mathbb{R}-c}(Y)$ one defines a pro-object of $D^b_{\mathbb{R}-c}(X;p_X)$ by setting

(2.2) $$\Phi^\mu_K(F) = "\varprojlim" Rq_{1!}(K_{X\times V} \otimes q_2^{-1}F)$$

where V runs over the set of relatively compact open subanalytic neighborhoods of $x = \pi(p)$. Actually one has the

LEMMA 2.2. — *For $K \in Ob\, D^b_{\mathbb{R}-c}(X \times Y)$ satisfying (2.1), this pro-object is an object of $D^b_{\mathbb{R}-c}(X;p_X)$ and the functor $\Phi^\mu_K : D^b_{\mathbb{R}-c}(Y;p_Y) \to D^b_{\mathbb{R}-c}(X;p_X)$ is well defined.*

Note that the functor $\Phi_K(\cdot) = Rq_{1!}(K \otimes q_2^{-1}(\cdot))$ would not be defined here in general.

PROPOSITION 2.3. — *Let $\varphi : (T^*Y)_{p_Y} \to (T^*X)_{p_X}$ be a germ of canonical transformation and Λ its associated germ of Lagrangian manifold in $T^*(X \times Y)$. One may find $K \in Ob\, D^b_{\mathbb{R}-c}(X \times Y)$ with $SS(K) \subset \Lambda$ in the neighborhood of (p_X, p_Y^a), such that $\Phi^\mu_K : D^b_{\mathbb{R}-c}(Y;p_Y) \to D^b_{\mathbb{R}-c}(X;p_X)$ is an equivalence of categories.*

3. – The category $D^b_{\mathbb{C}-c}(X;\Omega)$

Let now X be a complex n-dimensional manifold, and $X_{\mathbb{R}}$ the underlying real manifold. Recall that for $F \in Ob\, D^b_{\mathbb{R}-c}(X)$ one has the following characterisation (cf. [K-S 2]) :

$$(3.1) \qquad (F \in Ob\, D^b_{\mathbb{C}-c}(X)) \Longleftrightarrow (SS(F) \text{ is } \mathbb{C}^\times\text{-conical}) \Longleftrightarrow (SS(F) \text{ is } \mathbb{C}\text{-Lagrangian}),$$

thus we may define for any subset $\Omega \subset T^*X$ a full triangulated subcategory of $D^b_{\mathbb{R}-c}(X;\Omega)$ by setting

$$(3.2) \qquad \begin{cases} D^b_{\mathbb{C}-c}(X;\Omega) \underset{\text{def}}{=} \text{ the full subcategory of } D^b_{\mathbb{R}-c}(X;\Omega) \text{ of the objects} \\ F \in Ob\, D^b_{\mathbb{R}-c}(X) \text{ such that } SS(F) \text{ is } \mathbb{C}^\times\text{-conic in a neighborhood of } \Omega. \end{cases}$$

Proposition 3.1 (See [A, Appendix]). — *Let Y be another copy of X, $\varphi : (T^*Y)_{p_Y} \to (T^*X)_{p_X}$ be a germ of complex canonical transformation and $\Lambda \subset T^*(X \times Y)$ its associated complex Lagrangian submanifold. Then*

(i) *there exists $K \in Ob(D^b_{\mathbb{C}-c}(X \times Y;(p_X,p_Y^a)))$ with $SS(K) \subset \Lambda$ in a neighborhood of (p_X,p_Y^a) such that the functor of proposition 2.3 induces an equivalence of categories*

$$\Phi^\mu_K : D^b_{\mathbb{C}-c}(Y;p_Y) \to D^b_{\mathbb{C}-c}(X;p_X),$$

(ii) *if moreover φ is globally defined on the orbit $\mathbb{C}^\times p_Y$ then there is $K \in Ob(D^b_{\mathbb{C}-c}(X\times Y;\mathbb{C}^\times(p_X,p_Y^a)))$, with $SS(K) \subset \Lambda = \mathbb{C}^\times\Lambda$ in a neighborhood of $\mathbb{C}^\times(p_X,p_Y^a)$ such that Φ^μ_K induces an equivalence of categories*

$$\Phi^\mu_K : D^b_{\mathbb{C}-c}(Y;\mathbb{C}^\times p_Y) \to D^b_{\mathbb{C}-c}(X,\mathbb{C}^\times p_X).$$

Point (i) follows easily from proposition 2.3 by (3.1) because Φ^μ_K preserves local $\mathbb{C}^\times$-conicity, then (ii) stems from (i) and formula (2.2) that shows that Φ^μ_K is defined at any point in the fiber of π over $\pi(p)$.

For example one has $D^b_{\mathbb{C}-c}(X;T^*X) = D^b_{\mathbb{C}-c}(X)$ and if $x \in X \cong T^*_XX$ one has the equivalence $\Big(F \in Ob\, D^b_{\mathbb{C}-c}(X;x)\Big) \Longleftrightarrow \Big(F \in Ob\, D^b_{\mathbb{R}-c}(X)$ and $F|_V \in Ob\, D^b_{\mathbb{C}-c}(V)$ for some open neighborhood V of $x\Big)$.

Note that, in general the objects of $D^b_{\mathbb{C}-c}(X;p)$ do not have $\mathbb{C}$-constructible cohomologies and the natural functor $D^b_{\mathbb{C}-c}(X)/\mathcal{N}_p \cap D^b_{\mathbb{C}-c}(X) \to D^b_{\mathbb{C}-c}(X;p)$ is not an equivalence.

On the other hand, one has the following geometrical version of the generic position theorem. Recall (cf. [K-K]) that a complex Lagrangian subset $\Lambda \subset T^*X$ is said to have a *generic position* at $p \in \overset{\circ}{T^*} X$ iff

$$(3.3) \qquad \Lambda \cap \pi^{-1}\pi(p) = \mathbb{C}^\times p \text{ in a neighborhood of } p.$$

Proposition 3.2. — *Let $F \in Ob\, D^b_{\mathbb{C}-c}(X;p)$ such that $SS(F)$ is in a generic position at p. Then there exists $F' \in Ob\, D^b_{\mathbb{C}-c}(X;\pi(p))$ such that $F' \simeq F$ in $D^b(X;p)$.*

The proof goes by showing that one may "cut-off" the non $\mathbb{C}$-Lagrangian part of $SS(F)$ in $\pi^{-1}\pi(p)$, i.e. one finds kernels K, K^* in $D^b_{\mathbb{C}-c}(X \times X;(p,p^a))$ and an open subanalytic neighborhood U of x in X such that K, K^* satisfy the conditions of proposition 3.1 (i), $\Phi^\mu_{K^*}$ is a quasi-inverse of Φ^μ_K and $F' := \Phi^\mu_{K^*}((\Phi^\mu_K F)_U)$ is such that $SS(F')$ is $\mathbb{C}^\times$-invariant in $\pi^{-1}(U)$. Thus $F' \in Ob\, D^b_{\mathbb{C}-c}(X;\pi(p))$ by (3.1) and $F' \simeq F$ in $D^b(X;p)$ by proposition 3.1.

One may get a quicker proof by using a refined version, obtained in [D'A-S], of a microlocal cut-off lemma of [K-S 2] where one is allowed non-convex sets.

4. – Microlocalization of Perverse Sheaves

In [K-S 2] one finds the following microlocal characterisation of perverse sheaves :

On object $F \in Ob\, D^b_{\mathbb{C}-c}(X)$ is a perverse sheaf iff it satisfies the following condition

$$(4.1)\qquad \begin{cases} \text{For any non-singular point } p \in SS(F) \text{ such that } \pi : SS(F) \to X \\ \text{has constant rank in a neighborhood of } p, \text{ there exists a complex } d\text{-codimensional} \\ \text{submanifold } Y \subset X \text{ such that } F \simeq \mathbb{C}_Y[-d] \text{ in } D^b(X;p) \quad \text{(cf. [K-S 2, (10.3.7)]).} \end{cases}$$

Thus for any subset $\Omega \subset T^*X$ we may define a full subcategory $\mathrm{Perv}(X;\Omega)$ of $D^b_{\mathbb{C}-c}(X;\Omega)$ in the following manner.

DEFINITION 4.1. — $Ob\,\mathrm{Perv}(X;\Omega) \underset{\text{def}}{=} \{F \in Ob\, D^b_{\mathbb{C}-c}(X;\Omega)$; *F satisfies condition* (4.1) *at any p in a neighborhood of* $\Omega\}$.

Then the following results from §3 and the characterisation (4.1).

PROPOSITION 4.2. — *Let* $\Omega = \{p\}$ *(resp.* $\Omega = \mathbb{C}^\times p$*).*

(i) $\mathrm{Perv}(X;\Omega)$ *is invariant by extended canonical transformation in the sense of proposition 3.1 (i) (resp. proposition 3.1 (ii)).*

(ii) *Let* $F \in \mathrm{Perv}(X;p)$ *(resp.* $\mathrm{Perv}(X;\mathbb{C}^\times p)$*) such that* $SS(F)$ *is in a generic position at p. Then there is* $F' \in \mathrm{Perv}(X;\pi(p))$ *such that* $F \simeq F'$ *in* $D^b(X;p)$.

(iii) $\mathrm{Perv}(X;\Omega)$ *is a full abelian subcategory of* $D^b_{\mathbb{C}-c}(X;\Omega)$.

5. – The equivalence $\mathrm{Perv}(X;\mathbb{C}^\times p)^\circ \xrightarrow{\mu RH} \mathrm{Reghol}\,\mathcal{E}_{X,p}$

Recall that Kashiwara's functor RH of cohomology with bounds of [K 2, 3] is defined on $\mathbb{R}$-constructible complexes, more precisely

$$RH : D^b_{\mathbb{R}-c}(X)^\circ \to D^b(\mathcal{D}_X),$$

(where $D^b(\mathcal{D}_X)$ stands for $D^b(\mathrm{Mod}\,\mathcal{D}_X)$), and it is microlocalized in [A] as a functor

$$T\text{–}\mu hom(\cdot, \mathcal{O}_X) : D^b_{\mathbb{R}-c}(X)^\circ \to D^b_{\mathbb{R}_{>0}}(\pi^{-1}\mathcal{D}_X)\,,$$

where the latter category is the full subcategory subcategory of the complexes of $D^b(\pi^{-1}\mathcal{D}_X) := D^b(\mathrm{Mod}(\pi^{-1}\mathcal{D}_X))$ with $\mathbb{R}_{>0}$-homogenous cohomology. Since one has

$$\mathrm{supp}(T\text{–}\mu hom(F, \mathcal{O}_X)) \subset SS(F),$$

then for any subset $\Omega \subset T^*X$, the functor of triangulated categories

$$T\text{–}\mu hom(\cdot, \mathcal{O}_X) : D^b_{\mathbb{R}-c}(X;\Omega)^\circ \to D^b_{\mathbb{R}_{>0}}(\pi_\Omega^{-1}\mathcal{D}_X)$$

is well-defined, where $\pi_\Omega := \pi|_\Omega : \Omega \to X$. If moreover $\Omega = \mathbb{C}^\times\Omega$ is a $\mathbb{C}^\times$-invariant subset we set for $F \in Ob\, D^b_{\mathbb{R}-c}(X)$:

$$(5.1)\qquad \mu RH(F) \underset{\text{def}}{=} \gamma^{-1}R\gamma_*\, T\text{–}\mu hom(F, \mathcal{O}_X) \in Ob\, D^b_{\mathbb{R}_{>0}}(\pi_\Omega^{-1}\mathcal{D}_X).$$

Recall also the following facts

$$\text{For any } F \in Ob\, D^b_{\mathbb{R}-c}(X) \text{ and any } j \in \mathbb{Z},\ H^j T\text{–}\mu hom(F, \mathcal{O}_X) \text{ is an } \mathcal{E}_X^{\mathbb{R},f}\text{-module,}$$

$$\mathcal{E}_X^{\mathbb{R},f} \text{ is faithfully flat on } \mathcal{E}_X \text{ and } \gamma^{-1} R\gamma_* \mathcal{E}_X^{\mathbb{R},f} \cong \mathcal{E}_X\,,$$

and we have invariance by canonical transformations, that is, with the hypotheses of proposition 3.1 (i), one may find a section

$$s \in H^0(T\text{–}\mu hom(K, \Omega_{X\times Y/X}))_{(p_X, p_Y^a)},$$

(where $\Omega_{X\times Y/X}$ means the sheaf of maximum degree forms relative to $X \times Y \to X$) such that

$$\text{the correspondance } P \in \mathcal{E}^{\mathbb{R},f}_{X,p_X} \mapsto Q \in \mathcal{E}^{\mathbb{R},f}_{Y,p_Y} \text{ such that } Ps = sQ \text{ is a ring isomorphism}$$
$$\text{compatible with a natural isomorphism } T\text{–}\mu hom(F, \mathcal{O}_Y)_{p_Y} \xrightarrow{\sim} T\text{–}\mu hom(\Phi^{\mu}_{K[n]}F, \mathcal{O}_X)_{p_X}\,.$$

Finally we have a basic formula :

$$T\text{–}\mu hom(F, \mathcal{O}_X) \simeq \mathcal{E}_X^{\mathbb{R},f} \otimes_{\pi^{-1}\mathcal{D}_X} \pi^{-1} RH(F)\,, \qquad \text{for } F \in Ob\, D^b_{\mathbb{C}-c}(X)\,,$$

from which we get

$$\mu RH(F) = \mathcal{E}_X \otimes_{\pi^{-1}\mathcal{D}_X} \pi^{-1} RH(F) \qquad \text{for } F \in Ob\, D^b_{\mathbb{C}-c}(X)\,. \tag{5.2}$$

The key point is then the

LEMMA 5.1. — *Formula* (5.1) *actually defines a functor*

$$\mu RH : \mathrm{Perv}(X; \mathbb{C}^{\times} p)^{\circ} \to \mathrm{Reghol}(\mathcal{E}_{X,p}).$$

Proof : Let $F \in Ob\,\mathrm{Perv}(X; \mathbb{C}^{\times} p)$. By the invariance by extended (resp. quantized) canonical transformations, we may assume that $SS(F)$ has generic position at p, thus, by proposition 4.2 (iii) we may find $F' \in \mathrm{Perv}(X; \pi(p))$ such that $F \simeq F'$ in $D^b(X; p)$, thus

$$\mu RH(F)_p \simeq \mu RH(F')_p \simeq (\mathcal{E}_X \otimes_{\pi^{-1}\mathcal{D}_X} \pi^{-1} RH(F'))_p\,,$$

by (5.2), and the latter is an object concentrated in degree zero, which coincides with the germ at p of a regular holonomic $\mathcal{E}_X$-module.

♡

That $\mu RH : \mathrm{Perv}(X; \mathbb{C}^{\times} p)^{\circ} \to \mathrm{Reghol}(\mathcal{E}_{X,p})$ is an equivalence is then readily deduced, by using again invariance by canonical transformations, from Kashiwara and Kawai's generic position theorem of [K-K].

Details will appear elsewhere.

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